1.4 Exercises 4: Date 24.5.2013

1. We shall test the two wage equations of the last exercises now formally using the White test. Using the data of **cps78.xls** re-estimate the two equations

$$WAGE_i = \beta_0 + \beta_1 AGE_i + \beta_2 ED_i + \beta_3 FE_i + \beta_4 UNION_i + u_i$$

and

$$LNWAGE_i = \beta_0 + \beta_1 AGE_i + \beta_2 ED_i + \beta_3 FE_i + \beta_4 UNION_i + u_i.$$

To test for heteroscedasticity, calculate the residuals and save their squares e_i^2 of each equation. Calculate the squares and cross products of the explanatory variables. Now explain each series of the squared residuals by regressing on a constant, the original explanatory variables and their squares and cross products. E.g. for two explanatory variables x and y the test equation will be

$$e_i^2 = b_0 + b_1 x_i + b_2 y_i + b_3 x_i^2 + b_4 y_i^2 + b_5 x_i y_i + u_i.$$

Be careful to avoid multicollinearity! Under the null of homoscedasticity the number of observations (n) times the R^2 of the test equation is distributed χ_q^2 with q the number of regressors in the test equation minus one. Reject the null if nR^2 is larger than the critical level of the respective χ^2distribution.

2. Estimate the wage equation with WAGE as dependent variable with a heteroscedasticity consistent variance estimator (i.e. use the options in the regression software as e.g. in GRETL). Which differences do you observe?

3. Use the data in file **capm_usa.xls** to estimate the CAPM equation

$$R - R_f = \alpha + \beta(R_m - R_f) + u \tag{8}$$

for one firm of your choice. Check if the residuals of this equation are autocorrelated. Calculate the empirical autocorrelation function r_k of the residuals, i.e. calculate the empirical correlation of the residuals e_t with the residuals shifted by k Periods e_{t-k} for $k = 1, 2, ..., 20$ (see lecture notes p.28).

If the true $\rho_k = 0$ the estimated values should lie between $\pm\frac{2}{\sqrt{T}}$ with 95% probability. Plot k on the x-axis against r_k on the y-axis and draw the lines $\pm\frac{2}{\sqrt{T}}$. Is the assumption of uncorrelated errors for this model justified?

4. Now test the specific hypothesis of first order autocorrelation of the error term, i.e.

$$u_t = \rho u_{t-1} + \varepsilon_t \tag{9}$$

with ε_t a pure random term with mean $E(\varepsilon_t) = 0$ and constant variance $E(\varepsilon_t^2) = \sigma_\varepsilon^2$ for all t.
Compute the Durbin-Watson statistic for the residuals of equation (8).
Perform the DW-test by testing the following hypotheses:
a) $\rho > 0$, against the alternative $\rho \leq 0$.
b) $\rho < 0$, against the alternative $\rho \geq 0$.

5. Find a solution to the inhomogeneous first order difference equation (9) under the condition that $|\rho| < 1$ with a given disturbance function ε_t.

6. Assuming that ε_t is a pure random term with mean $E(\varepsilon_t) = 0$ and constant variance $E(\varepsilon_t^2) = \sigma_\varepsilon^2$ for all t, show that for equation (9) under the condition that $|\rho| < 1$, the variance of u is equal to

$$E(u_t^2) = \frac{\sigma_\varepsilon^2}{1 - \rho^2} \tag{10}$$

   Use the answer to question 5 in the derivation.

7. Show that for equation (9) under the condition that $|\rho| < 1$ and the assumption of question 6, the covariance between u_t and u_{t-1} is equal to

$$E(u_t u_{t-1}) = \frac{\rho\sigma_\varepsilon^2}{1 - \rho^2}. \tag{11}$$

8. What is the consequence of having autocorrelation or heteroskedasticity in residuals for the statistical properties of the ordinary least squares estimator?

9. Estimate the CAPM equation for the firm DEC and test if you can detect a structural break in March 1982 by using the Chow-test.

10. Re-estimate the CAPM equation for the firm DEC using the observations from 1978:1 until March 1982 (51 observations). Test if the property of Beta is the same as when using the full set of observations.