1.5 Exercises 5: Date 26.6.2015

1. Derive the mean $\mu = E(y_t)$ and the variance $\gamma_0 = E(y_t - \mu)^2$ and the autocovariances $\gamma_j = E[(y_t - \mu)(y_{t-j} - \mu)]$ for $j = 1, 2, 3$, of the process $y_t = 0.8y_{t-1} + u_t$. (See lecture notes p.55)

2. Generate 100 observations of an independent normal random variable $u_t \sim N(0, 1)$ and define a new random variable $y_t = 0.8y_{t-1} + u_t$. Use an initial value of $y_0 = 1$. Produce a time plot of the series y_t. Is this a stationary process?

3. Use the generated series to calculate the mean, the variance and the autocorrelation function of y_t up to third order using the sample autocorrelation coefficients

$$r_k = \frac{\sum_{t=k+1}^{T} (y_t - \bar{y})(y_{t-k} - \bar{y})}{\sum_{t=1}^{T} (y_t - \bar{y})^2} \tag{11}$$

   For the mean always use the sample mean $\bar{y} = \frac{1}{T} \sum_{t=1}^{T} y_t$. Describe the pattern of the estimated ACF. Compare the property of the ACF to your results of question 1.

4. Calculate the PACF for the y_t series by running three regressions $y_t = \beta_0 + \beta_1 y_{t-1} + ... + \beta_p y_{t-p} + u_t$ for $p = 1, ..., 3$. The sequence of regression coefficients of variable y_{t-p} for $p = 1, ..., 3$ is the PACF.

5. Calculate the PACF for the y_t series using the Yule Walker equations for $p = 1, ...3$ and compare the result with your answer to question 4. What is the theoretical PACF of the process $y_t = 0.8y_{t-1} + u_t$.

6. What kind of information can be obtained from the estimated ACF and PACF that would be relevant for the specification of a time series model?

7. Derive the mean, the variance and the autocovariance function of order up to 3 of the process $y_t = 0.5 + u_t + 0.6u_{t-1}$. Is this process stationary?

8. Generate 100 observations of an independent normal random variable $u_t \sim N(0, 1)$ and define a new random variable $y_t = 0.5 + u_t + 0.6u_{t-1}$. Use $u_0 = 0.1$ as starting value. Calculate the ACF of this process and compare with your answer to question 7.

9. Produce a vector of 100 realisations of independent normal random variables $u_t \sim N(0, 1)$. Compute the values of variable x_t as $x_t = 2 + x_{t-1} + u_t$ for $t = 1, ..., 100$ with a given initial value $x_0 = 5$. Produce another vector of 100 realisations of independent normal random variables $\varepsilon_t \sim N(0, 1)$. (Check that the random number generator generates different values than before). Compute the time series $y_t = 1 + y_{t-1} + \varepsilon_t$ with initial value $y_0 = 2$. Then estimate the model with the 100 observations of x_t and y_t

$$y_t = a + bx_t + \xi_t \tag{12}$$

   by the least squares method. Compute R^2 and test the parameter b against zero. How do you interpret the result?

10. If variables x_t und y_t are really dependent on each other this should also hold for the differences of these variables. Thus, estimate the model in differences

$$y_t - y_{t-1} = c + d(x_t - x_{t-1}) + \zeta_t. \tag{13}$$

by least squares What is your result? Test parameter d against zero. How do you explain this result?