2. Übung Mathematische Statistik WS15

In the following problems, use level $\alpha = 0.05$.

1. For a sample of size twenty from an alternative distribution, calculate the uniformly optimal test for $H_0 : p \leq 0.4$ against $H_1 : p > 0.4$.

2. For a sample of size twenty from an alternative distribution, calculate the uniformly optimal test for $H_0 : p \geq 0.4$ against $H_1 : p < 0.4$.

3. For a sample of size twenty from an alternative distribution, calculate the uniformly optimal unbiased test for $H_0 : p = 0.4$ against $H_1 : p \neq 0.4$.

4. For a sample of size twenty from an alternative distribution, calculate the likelihood ratio test for $H_0 : p = 0.4$ against $H_1 : p > 0.4$.

5. For a sample of size twenty from an alternative distribution, calculate a randomized test for $H_0 : p \leq 0.4$ against $H_1 : p > 0.4$ with symmetric probabilities (i.e., the probability that $\hat{p} > 0.4$ and H_0 is rejected equals $\alpha/2$).

6. For a sample from an alternative distribution and for large sample size n, the distribution of $\hat{p}$ can be approximated by a normal distribution.

   (a) Use this fact to construct an approximate test for the two-sided alternative.

   (b) Use the above and the relationship between tests and confidence intervals to construct an approximate confidence interval for p.

7. In a sample of 200 products, 15 defective items were found. The manufacturer claims that the percentage of defective items is at most 5%. Test at the 5% level.

8. A sample of 60 bottles of beer showed a mean content of 470 ccm with a sample variance of 2000. Assuming a normal distribution, test $H_0 : \mu = 500$.

9. The lifetime of lightbulbs is supposed to be exponentially distributed. A certain brand is claimed to have an expected lifetime of 2000 hours. A sample showed the following lifetimes:

   900 1760 2140 2050 2820 2200 3000 600 1500 1980

   Test at the 5% level.

10. For the following sample from a normal distribution

    1.1 0.7 1.6 1.3 0.8 1.4 0.7 0.6 1.8 1.4 1.3

    test $H_0 : \mu = 1.0$ against the two-sided alternative, if $\sigma^2 = 0.5$

11. Do the previous problem for unknown variance.

12. For the sample in problem 10, test whether $\sigma^2 = 0.5$ ($\alpha = 0.1$).