4. Übung Höhere Wahrscheinlichkeitstheorie

1. Let ℓ be a nonnegative positively homogeneous additive functional on

$$C_b^+(X) = \{f \in C_b(X) : f \geq 0\}$$

(i.e., $\ell(f+g) = \ell(f) + \ell(g)$ and for $c \geq 0$ $\ell(cf) = c\ell(f)$). Show that $\ell(f-g) = \ell(f) - \ell(g)$ defines a positive linear functional on $C_b(X)$.

2. Prove that a positive linear functional on $C_b(X)$ is continuous and that a positive linear functional on $C_c(X)$ is continuous iff the associated Radon measure is finite.

3. Assume that X is compact. Show hat a continuous linear functional ℓ on $C(X)$ is positive iff $\ell(1) = \|\ell\|$.

4. Let μ be a finite Radon measure on a metric space X. Show that there is a separable closed subset of X that has full measure.

5. Prove the Lévy-Ottaviani inequality:

   Let $\xi_1, \ldots, \xi_n$ be independent, $a, b > 0$, and $S_n = \sum_{i=1}^n \xi_i$. Then

$$\mathbb{P}(\max_{i \leq n} |S_i| \geq a + b) \leq \frac{\mathbb{P}(|S_n| \geq a)}{1 - \max_{i \leq n} \mathbb{P}(|S_n - S_i| \geq b)}.$$