Exercise 2 – gravimetry

Assuming that a spherical cavity (radius = 8m) is filled with water and its center is located 15 m below the surface, as presented in Figure 1. Please, use matlab to plot the gravitational force that would be observed at the surface if measurements are performed on the surface with 0.5 m separation between stations.

The surrounding rocks are Shales (density ~2400 kg/m^3).

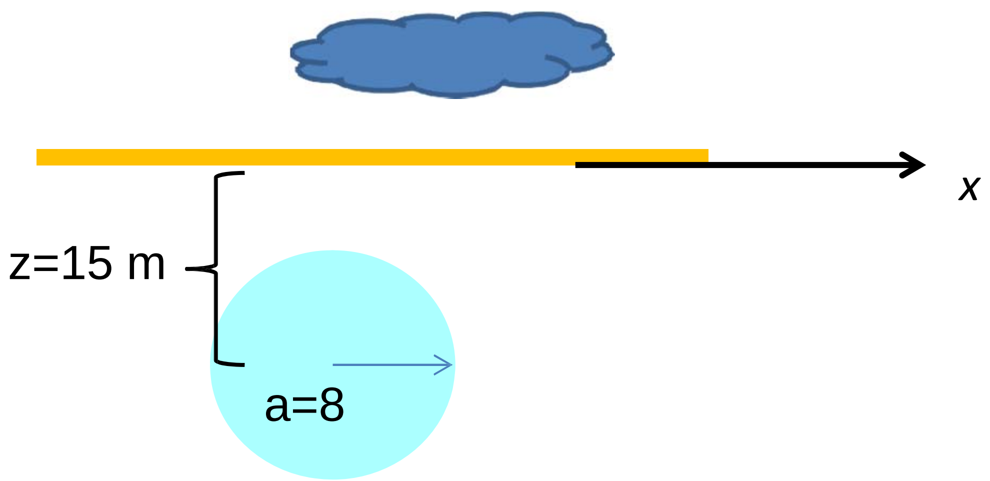

Figure 1

The analytical model to compute the vertical component of the gravity effect (*Δg*) due to a sphere of constant density is given by:

$$\Delta g(x) = G\frac{\Delta M}{r^2}\frac{h}{r} = G\frac{\Delta M}{z^2+x^2}\frac{z}{(z^2+x^2)^{1/2}} \qquad (1)$$

where, *ΔM* is the difference in the mass from the target (sphere) and the surrounding material; *r* is the distance to the center of the Earth; *z* the distance (depth) between the center of the target (i.e., sphere) and the surface; *x* is the distance along profile direction, and *a* is the radius of the sphere.

1. Compute the mass excess (or deficiency) for the given target, i.e., the water sphere;

2. Implement in matlab the analytical equation (1) used to compute the gravitational effect of a sphere.

3. Plot the gravitational effect due to the sphere represented in Figure 1, for measurements performed in the surface (every 0.5 m).

4. Plot the gravitational effect due to the same sphere but for the center of the sphere located at a depth of 40 m (z=40).

5. Plot a line (at the y-axis) to indicate the half of the maximum gravitational effect (Δg /2)

6. Find the distance along profile direction (x), at which the $\Delta g/2$ is located. This value is commonly referred to as the half width ($x_{1/2}$)

7. Use the analytical approximation ($z = 1.3\ x_{1/2}$) to compute the depth of a sphere (from the Earth surface to the center of the sphere) and compare its value with the kwon depth.

8. If the computed depth (Step 7) is not the same as the original value, discuss possible reasons.

9. Compute the analytical response of a sphere as illustrated in Figure 1, but assuming that it is filled with gold instead of water. The density of gold is 19300 (kg/m^3). Plot the analytical gravity curve for the sphere filled with water and filled with gold. Discuss the observed response.

10. Compute the analytical gravity curve for a sphere filled with gold (similar to step 9) but placed at a depth of 70 m and for the case of a radius of 20m and for 5m radius (a=20 and a=5). Discuss the results.

Hint: If you have a curve ***D*** as a function of ***x***, i.e., ***D***(***x***), and you want to find in the value of ***x*** at which a given value of ***D*** is given, then you can use in matlab the following script:

```
X=-200:0.5:200;

idx = find(diff(D >= yval));

xval = x(idx) + (yval - D(idx)) .* (x(idx+1) -
x(idx)) ./ (D(idx+1)) - D(idx);
```

yval - is the value of function ***D*** that you know

idx – the index pointing to the ***yval*** in the variable representing the ***x*** vector

xval – the unknown ***x***-value (at the ***x***-coordinate) at which the known ***yval*** is found